Mathematics and Implementation Details for the Equal Earth Projection

The Equal Earth projection is an equal-area pseudocylindrical projection. It is appropriate for mapping global phenomena or for any other thematic world map that requires areas at their true relative sizes. Its key features are its resemblance to the popular Robinson projection and continents with a visually pleasing appearance similar to those found on a globe.

The research paper on the Equal Earth projection is available here:
https://www.tandfonline.com/doi/full/10.1080/13658816.2018.1504949

I. Forward equations

To derive the projected coordinates of a point, geodetic latitude (ϕ) is first converted to authalic latitude (β) and a radius of authalic sphere (R_A) that has same surface area as an ellipsoid (like WGS 1984) is calculated (after Snyder (1987)). The formulae from geodetic longitude and latitude (λ, ϕ) to the projected coordinates (x, y) are:

$$q_P = (1 - e^2)\left[\frac{1}{1-e^2} - \frac{1}{2e} \cdot \ln\left(\frac{1-e}{1+e}\right)\right]$$

$$q = (1 - e^2)\left[\frac{\sin\phi}{1-e^2 \cdot \sin^2\phi} - \frac{1}{2e} \cdot \ln\left(\frac{1-e \cdot \sin\phi}{1+e \cdot \sin\phi}\right)\right]$$

$$\beta = \operatorname{asin}\left(\frac{q}{q_P}\right) \text{ and } R_A = a \cdot \sqrt{\frac{q_P}{2}}$$

$$\sin\theta = \frac{\sqrt{3}}{2} \cdot \sin\beta$$

$$x = \frac{R_A \cdot 2\sqrt{3} \cdot \lambda \cdot \cos\theta}{3 \cdot \left(A_1 + 3 \cdot A_2 \cdot \theta^2 + \theta^6 \cdot (7 \cdot A_3 + 9 \cdot A_4 \cdot \theta^2)\right)}$$

$$y = R_A \cdot \theta \cdot \left(A_1 + A_2 \cdot \theta^2 + \theta^6 \cdot (A_3 + A_4 \cdot \theta^2)\right)$$

where:

- x and y are the projected coordinates,
- λ and ϕ are the geodetic longitude and the latitude,
- λ_0 is the central meridian,
- a and e are the semi-major axis and eccentricity of the ellipsoid,
- β and R_A are derived authalic latitude and radius of authalic sphere,
- θ is the parametric latitude, and
- A_1 to A_4 are polynomial coefficients defined as:
  - $A_1 = 1.340264$, $A_2 = -0.081106$, $A_3 = 0.000893$, and $A_4 = 0.003796$.

II. Reverse equations

The reverse conversion from the projected coordinates (x, y) to geodetic longitude and latitude (λ, ϕ) requires iteration of y equation to obtain the parametric latitude θ. We suggest using the Newton-Raphson method, where y/R_A is taken as a first trial θ_0, a correction $\Delta\theta_0$ is calculated and subtracted from preceding trial θ_0 to obtain the next trial θ_1:

$$\theta_0 = \frac{y}{R_A} \qquad \Delta\theta_0 = \frac{\theta_0 \cdot \left(A_1 + A_2 \cdot {\theta_0}^2 + {\theta_0}^6 \cdot \left(A_3 + A_4 \cdot {\theta_0}^2\right)\right) - \frac{y}{R_A}}{\left(A_1 + 3 \cdot A_2 \cdot {\theta_0}^2 + {\theta_0}^6 \cdot \left(7 \cdot A_3 + 9 \cdot A_4 \cdot {\theta_0}^2\right)\right)}$$

$$\theta_1 = \theta_0 - \Delta\theta_0 \qquad \Delta\theta_1 = \frac{\theta_1 \cdot \left(A_1 + A_2 \cdot {\theta_1}^2 + {\theta_1}^6 \cdot \left(A_3 + A_4 \cdot {\theta_1}^2\right)\right) - \frac{y}{R_A}}{\left(A_1 + 3 \cdot A_2 \cdot {\theta_1}^2 + {\theta_1}^6 \cdot \left(7 \cdot A_3 + 9 \cdot A_4 \cdot {\theta_1}^2\right)\right)}$$

$$\theta_2 = \theta_1 - \Delta\theta_1 \qquad \text{etc.}$$

The calculation is repeated until $\Delta\theta_n$ is less than a predetermined convergence value ϵ. Then, using the final θ_{n+1} as θ, the geodetic longitude and latitude (λ, ϕ) of a point are determined as follows:

$$\lambda = \lambda_0 + \frac{x \cdot \sqrt{3} \cdot \left(A_1 + 3 \cdot A_2 \cdot \theta^2 + \theta^6 \cdot \left(7 \cdot A_3 + 9 \cdot A_4 \cdot \theta^2\right)\right)}{2 \cdot R_A \cdot \cos\theta}$$

$$\sin\beta = \frac{2}{\sqrt{3}} \cdot \sin\theta$$

$$\phi = \beta + \left(\frac{1}{3}e^2 + \frac{31}{180}e^4 + \frac{517}{5040}e^6 + \cdots\right)\sin 2\beta + \\ + \left(\frac{23}{360}e^4 + \frac{251}{3780}e^6 + \cdots\right)\sin 4\beta + \left(\frac{761}{45360}e^6 + \cdots\right)\sin 6\beta$$

where:

- x and y are the projected coordinates,
- λ and ϕ are the geodetic longitude and the latitude,
- λ_0 is the central meridian,
- e is the eccentricity of the ellipsoid,
- R_A is the radius of authalic sphere defined as in forward equations,
- β is the authalic latitude,
- θ is the parametric latitude,
- θ_0 to θ_{n+1} and $\Delta\theta_0$ to $\Delta\theta_n$ are trials and corrections for the parametric latitude θ in the Newton-Raphson iteration method, and
- A_1 to A_4 are polynomial coefficients defined as in forward equations.

III. Equations for the sphere Earth models

When a sphere used as Earth's model instead of an ellipsoid, the forward and reverse equations simplify with:

$\beta = \varphi$ and $R_A = R$

where:

φ is the latitude on the sphere, and

R is the radius of the sphere.

IV. Testing example

Parameters:

Ellipsoid: WGS 1984 $a = 6378137.0\ m$

$e = 0.08181919084262$

$R_A = 6371007.181\ m$

Central meridian: $\lambda_0 = 90°\ W$

Input point: $\lambda = 117°11'48.349"\ W$

$\phi = 34°03'27.169"\ N$

Forward: $\beta = 0.592339964$

$\theta = 0.504654838$

$\boldsymbol{x = -2390749.043\ m}$

$\boldsymbol{y = 4242849.758\ m}$

Reverse Newton-Raphson iteration:

Step 1: $\theta_0 = 0.665962169$ $\Delta\theta_0 = 0.164312383$

Step 2: $\theta_1 = 0.501649786$ $\Delta\theta_1 = -0.003004201$

Step 3: $\theta_2 = 0.504653987$ $\Delta\theta_2 = -8.512742 \cdot 10^{-7}$

Step 4: $\theta_3 = 0.504654838$ $\Delta\theta_3 = -6.859963 \cdot 10^{-14}$

$\theta = \theta_4 = 0.504654838$

Then: $\beta = 0.592339965$

$\boldsymbol{\lambda = 117°11'48.349"\ W}$

$\boldsymbol{\phi = 34°03'27.169"\ N}$

Reference:

Snyder, J. P. 1987. Map Projections–A working manual. Professional Paper 1395. US Geological Survey, Washington, DC.

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